arXiv:1003.3507v2 [cs.IT] 22 Mar 2010

On the Degrees of Freedom Regions of Two-User MIMO Z and Full Interference Channels with Reconfigurable Antennas

Lei Ke and Zhengdao Wang
Dept. of ECE, Iowa State University, Ames, IA 50011, USA.
Email: {kelei, zhengdao}@iastate.edu

***Abstract*—We study the degrees of freedom (DoF) regions of two-user multiple-input multiple-output (MIMO) Z and full interference channels in this paper. We assume that the receivers always have perfect channel state information. We derive the DoF region of Z interference channel with channel state information at transmitter (CSIT). For full interference channel without CSIT, the DoF region has been obtained in previous work except for a special case $M_1 < N_1 < \min(M_2, N_2)$, where M_i and N_i are the number of transmit and receive antennas of user i, respectively. We show that for this case the DoF regions of the Z and full interference channels are the same. We establish the achievability based on the assumption of transmitter antenna mode switching. A systematic way of constructing the DoF-achieving nulling and beamforming matrices is presented in this paper.**

***Index Terms*—Degrees of freedom region, Z interference channel, interference channel, multiple-input multiple-output.**

I. Introduction

Characterizing capacity region of interference channel has been a long open problem. Many researchers investigated this important area, and the capacity regions of certain interference channels are known when the interference is strong, e.g. [1]–[3]. However, when the interference is not strong, the capacity region is still unknown. Recent progress reveals the capacity region for two-user interference channel within one bit [4], and after that the capacity region for very weak interference channel is settled [5]–[7]. Recently, a deterministic channel model has been proposed and used to explore the capacity of Gaussian interference network [8]–[10] such that the gap to capacity region can be bounded up to a constant value.

When it comes to multiple-input multiple-output (MIMO) networks, the capacity regions of certain MIMO interference channels are known [11], [12]. Instead of trying to characterize the capacity region completely, the degrees of freedom (DoF) region characterizes how capacity scales with transmit power as the signal-to-noise ratio goes to infinity.

It is well-known that in certain cases, the absence of channel state information at transmitter (CSIT) will not affect the DoF for MIMO networks, e.g., in the multiple access channel [13]. In other cases, CSIT does play an important role. For example, using interference alignment scheme, it is shown that the total DoF of a K-user MIMO interference channel can be $MK/2$, where M is the number of antennas of each user [14]. The key idea is to pack interferences from multiple sources to reduce the dimensionality of signal space spanned by interference.

The DoF region of two-user MIMO interference channel with CSIT has been obtained [15], where it is shown that zero forcing is enough to achieve the DoF region. However, it is a different story in two-user MIMO X channel, where each transmitter has a message to every receiver. In [16] it is shown that interference alignment is the key to achieving the DoF region of MIMO X network. The DoF region of two-user MIMO broadcast channel and interference channel without CSIT are considered in [17], where there is an uneven trade-off between the two users. Except for a special case, the DoF region for the interference channel is known and achievable. Similar, but more general result of isotropic fading channel can be found in [18]. The DoF regions of the K-user MIMO broadcast, interference and cognitive radio channels are derived in [19] for some cases. However, the special case in [17] remains unsolved.

When only one of the two transmitter-receiver pairs is subject to interference, then the interference channel is termed as Z interference channel (ZIC). To avoid confusion, we will call the channel where both pairs are subject to interference the full interference channel (FIC). The capacity regions of MIMO Gaussian ZIC is established in [20] under very strong interference and aligned strong interference cases. In [21], the authors considered the capacity region of a single antenna ZIC without CSIT using deterministic approach.

Recently, it is shown in [22] that if the channel is staggered block fading, we can explore the channel correlation structure to do interference alignment, where

the upper bound in the converse can be achieved in some special cases. For example, it is shown that for two-user MIMO staggered block fading FIC with 1 and 3 antennas at transmitters, 2 and 4 antennas at their corresponding receivers and without CSIT, the DoF pair $(1, 1.5)$ can be achieved. The idea was further clarified in [23], where a blind interference alignment scheme is also proposed for K-user multiple-input single-output (MISO) broadcast channel to achieve DoF outer bound when CSIT is absent. Their approach is to use reconfigurable antennas such that the antennas can be switched among different modes to artificially create channel variation. It is their work that inspires us to investigate the remaining unknown case in the two-user MIMO FIC.

In this paper, we investigate the DoF region of MIMO ZIC with or without CSIT. We found that for the unknown cases of MIMO FIC it is enough to consider an equivalent ZIC, for which we propose a joint beamforming and nulling scheme to achieve its converse upper bound. We assume that perfect CSI is always available at receivers and transmitter of one user is capable of antenna mode switching. Based on this, we describe the DoF regions of MIMO two-user ZIC and FIC.

We first present the system model in Section II. The known results on the DoF region of two-user MIMO FIC are briefly reviewed in Section III. We present the exact DoF regions of ZIC and FIC in Section IV. Finally Section V concludes our results.

Notation: boldface uppercase letters denote matrices or vectors. $\mathbb{R}, \mathbb{C}$ are the real and complex numbers sets. $\mathcal{CN}(0, 1)$ denotes a circularly symmetric complex normal distribution with zero mean and unit variance. We use $\boldsymbol{A} \otimes \boldsymbol{B}$ to denote the Kronecker product between $\boldsymbol{A}$ and $\boldsymbol{B}$. $\mathbf{0}$ and $\mathbf{1}$ denote all one and all zero vectors, respectively. $\boldsymbol{A}^T$ and $\boldsymbol{A}^H$ denote the transpose and Hermitian of $\boldsymbol{A}$, respectively. We also use notation like $\boldsymbol{A}_{m\times n}$ to emphasize that $\boldsymbol{A}$ is of size $m \times n$. We use $\boldsymbol{I}_m$ to denote a size $m \times m$ identity matrix.

II. System Model

Consider a MIMO interference channel with two transmitters and two receivers, the number of transmit (receive) antennas at the ith transmitter (receiver) is denoted as M_i (N_i), $i \in \{1, 2\}$. The system is termed as an (M_1, N_1, M_2, N_2) system, which can be described as

$$\boldsymbol{Y}_1(t) = \boldsymbol{H}_{11}(t)\boldsymbol{X}_1(t) + \boldsymbol{H}_{12}(t)\boldsymbol{X}_2(t) + \boldsymbol{Z}_1(t) \quad (1)$$

$$\boldsymbol{Y}_2(t) = \boldsymbol{H}_{21}(t)\boldsymbol{X}_1(t) + \boldsymbol{H}_{22}(t)\boldsymbol{X}_2(t) + \boldsymbol{Z}_2(t) \quad (2)$$

where t is the time index, $\boldsymbol{Y}_i(t) \in \mathbb{C}^{N_i}$, $\boldsymbol{Z}_i(t) \in \mathbb{C}^{N_i}$ are the received signal and additive noise of receiver i, respectively. The entries of $\boldsymbol{Z}_i(t)$ are independent and identically $\mathcal{CN}(0, 1)$ distributed in both time and space. The channel between the ith transmitter and the jth receiver is denoted as $\boldsymbol{H}_{ji}(t) \in \mathbb{C}^{N_j\times M_i}$. We assume that the probability of $\boldsymbol{H}_{ij}(t)$ belonging to any subset of $\mathbb{C}^{N_j\times M_i}$ that has zero Lebesgue measure is zero. For the two-user MIMO ZIC, $\boldsymbol{H}_{21}(t) = 0$. $\boldsymbol{X}_i(t) \in \mathbb{C}^{M_i}$ is the input signal at transmitter i and $\boldsymbol{X}_1(t)$ is independent of $\boldsymbol{X}_2(t)$. The transmitted signals satisfy the following power constraint:

$$\mathrm{E}(||\boldsymbol{X}_i(t)||^2) \le P \quad i = 1, 2 \quad (3)$$

Denote the capacity region of the two-user MIMO system as $C(P)$, which contains all the rate pairs (R_1, R_2) such that the corresponding probability of error can approach zero as coding length increases. The DoF region is defined as follows [17]

$$\mathcal{D} := \Big\{(d_1, d_2) \in \mathbb{R}_2^+ : \exists(R_1(P), R_2(P)) \in C(P), \text{ such that } d_i = \lim_{P\to\infty} \frac{R_i(P)}{\log(P)}, \quad i = 1, 2\Big\}.$$

III. Known Results on DoF Region of MIMO Full Interference Channel

We first present some known results on DoF region of MIMO full interference channel which is useful for developing our results.

The degrees of freedom region of two-user MIMO full interference channel with CSIT is the following [15, Theorem 2]:

$$d_i \le \min(M_i, N_i), \quad i = 1, 2; \quad (4)$$

$$d_1 + d_2 \le \min(\max(N_1, M_2), \max(M_1, N_2), N_1 + N_2, M_1 + M_2) \quad (5)$$

An outer bound of degrees of freedom region of two-user MIMO full interference channel without CSIT is as follows [18, Theorem 1]:

$$\text{for } i = 1, 2, \quad d_i \le \min(M_i, N_i); \quad (6)$$

$$d_1 + \frac{\min(N_1, N_2, M_2)}{\min(N_2, M_2)} d_2 \le \min(M_1 + M_2, N_1); \quad (7)$$

$$\frac{\min(N_1, N_2, M_1)}{\min(N_1, M_1)} d_1 + d_2 \le \min(M_1 + M_2, N_2). \quad (8)$$

Note that the same result is also given in [17], though in a less compact form.

It is known [17] that when $N_1 < N_2$, the outer bound given in (6), (7) and (8) can be achieved by zero forcing or time sharing except for the case $M_1 < N_1 < M_2$, for which we do not know how to achieve

$$(d_1, d_2) = (M_1, \frac{\min(M_2, N_2)(N_1 - M_1)}{N_1}) \quad (9)$$

in general.

The outer bound given in [18, Theorem 1] is derived based on the assumption that i) $\boldsymbol{H}_{12}$ and $\boldsymbol{H}_{22}$

are statistically equivalent, and ii) $\boldsymbol{H}_{21}$ and $\boldsymbol{H}_{11}$ are statistically equivalent. Since these assumptions still hold when antenna mode switching is used at transmitters, the DoF outer bound specified by (6)–(8) is still valid.

IV. DoF Regions of MIMO ZIC and FIC

In this section we will first discuss the DoF regions of MIMO ZIC with or without CSIT, the extension to MIMO FIC will be made at the end of this section.

A. Two-User MIMO ZIC with CSIT

Theorem 1: The degrees of freedom region of two-user MIMO Z interference channel with CSIT is the following

$$
\begin{aligned}
d_i &\le \min(M_i, N_i), \quad i = 1, 2, \\
d_1 + d_2 &\le \min(\max(N_1, M_2), N_1 + N_2, M_1 + M_2).
\end{aligned}
$$

Proof: The theorem can be proven based on the result in [15]. We omit the proof due to space limit. ■

B. Two-User MIMO ZIC without CSIT

Lemma 1: The outer bound of degrees of freedom region of two-user MIMO Z interference channel without CSIT can be given as

$$
d_i \le \min(M_i, N_i), \quad i = 1, 2 \tag{10}
$$

$$
d_1 + \frac{\min(N_1, N_2, M_2)}{\min(N_2, M_2)} d_2 \le \min(M_1 + M_2, N_1). \tag{11}
$$

Proof: This is the direct result of [18, Theorem 1] by noticing that there is no interference from transmitter 1 to receiver 2 hence (8) is not longer needed. ■

Lemma 2: For the two-user MIMO Z interference channel, when $N_1 \ge N_2$, (11) is achievable by zero forcing.

Proof: This can be shown by noticing that (11) is reduced to $d_1+d_2 \le \min(M_1+M_2, N_1)$ and zero forcing suffices. ■

We have the following lemma regarding the relationship between DoF regions of ZIC and FIC.

Lemma 3: When $N_1 \le N_2$, the MIMO ZIC and FIC have the same DoF regions. Any encoding scheme that is DoF optimal for one channel is also DoF optimal for the other.

Proof: Any point in the FIC is also trivially achievable in the ZIC because user 2's channel is interference free. Conversely, any point achievable in the ZIC region, is also achievable in FIC. This is based on the fact that the channels are statistically equivalent at both receivers. If receiver 1 can decode user 1's message, then receiver 2, having at least as many antennas, must also be able to decode the same message. Receiver 2 can then subtract the decoded message, which renders the resulting channel the same as in the ZIC. ■

Due to Lemma 3, we can translate all achievability schemes from FIC to ZIC in a trivial way. However, for the case $M_1 < N_1 < \min(M_2, N_2)$, there is still no DoF-optimal encoding scheme for the two-user MIMO FIC.

In the following, we develop a scheme to achieve (9) for MIMO ZIC, which is our main result of this paper.

Consider the two-user MIMO ZIC with $M_1 < N_1 < M_2 = N_2$. We want to show that the following DoF pair is achievable

$$
(d_1, d_2) = \left(M_1, \frac{M_2(N_1 - M_1)}{N_1} \right). \tag{12}
$$

We notice that this point can not be achieved by zero forcing over one time instant. This is because using zero forcing if transmitter 1 sends M_1 streams, transmitter 2 can only send $N_1 - M_1$ streams without interfering receiver 1. If transmitter 2 sends more streams, the desired signal and interference are not separable at receiver 1 as transmitter 2 does not know channel state information so it can not send streams along the null space of $\boldsymbol{H}_{12}$. A simple example is the $(1, 2, 3, 3)$ case, where the outer bound gives us $(1, 1.5)$, which is not achievable via zero forcing over one time slot.

We make the assumption that the channel $\boldsymbol{H}_{12}$ stays the same for at least N_1 time slots, and there are N_1 antenna modes transmitter 1 can use. It is sufficient to show that $(M_1 N_1, M_2(N_1 - M_1))$ streams can be achieved in N_1 time slots. The transmitter 1 will use different antenna modes to create channel variation. The time expansion channel between transmitter 1 and receiver 1 will be

$$
\tilde{\boldsymbol{H}}_{11} = \begin{bmatrix} \boldsymbol{H}_{11}(1) & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{H}_{11}(2) & \boldsymbol{0} & \boldsymbol{0} \\ \vdots & \vdots & \ddots & \vdots \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{H}_{11}(N_1) \end{bmatrix}_{N_1^2 \times N_1 M_1}
$$

and the channel between transmitter 2 and receiver 1 is

$$
\tilde{\boldsymbol{H}}_{12} = \boldsymbol{I}_{N_1} \otimes \boldsymbol{H}_{12}(1) \tag{13}
$$

as user 2 does not create channel variation. Here and after, we use tilde notation to indicate the time expansion signals. We will use precoding at transmitter 2 only and nulling at receiver 1 only. Let $\tilde{\boldsymbol{P}}$ be the transmit beamforming matrix at transmitter 2 and $\tilde{\boldsymbol{Q}}$ be the nulling matrix at receiver 1. We propose to use the following structures for them

$$
\tilde{\boldsymbol{P}}_{M_2 N_1 \times M_2(N_1 - M_1)} = \boldsymbol{P}_{N_1 \times (N_1 - M_1)} \otimes \boldsymbol{I}_{M_2} \tag{14}
$$

$$
\tilde{\boldsymbol{Q}}_{M_1 N_1 \times N_1^2} = \boldsymbol{Q}_{M_1 \times N_1} \otimes \boldsymbol{I}_{N_1}. \tag{15}
$$

The received signal at receiver 1 can be written as

$$
\tilde{\boldsymbol{Y}}_1 = \tilde{\boldsymbol{H}}_{11} \tilde{\boldsymbol{X}}_1 + \tilde{\boldsymbol{H}}_{12} \tilde{\boldsymbol{P}} \tilde{\boldsymbol{X}}_2 + \tilde{\boldsymbol{Z}}_1, \tag{16}
$$

where $\tilde{\boldsymbol{X}}_1$ is a length M_1N_1 vector, and $\tilde{\boldsymbol{X}}_2$ is a length $M_2(N_1-M_1)$ vector. After applying nulling matrix $\tilde{\boldsymbol{Q}}$ we have

$$\tilde{\boldsymbol{Q}}\tilde{\boldsymbol{Y}}_1 = \underbrace{\tilde{\boldsymbol{Q}}\tilde{\boldsymbol{H}}_{11}}_{\tilde{\boldsymbol{U}}}\tilde{\boldsymbol{X}}_1 + \underbrace{\tilde{\boldsymbol{Q}}\tilde{\boldsymbol{H}}_{12}\tilde{\boldsymbol{P}}}_{\tilde{\boldsymbol{V}}}\tilde{\boldsymbol{X}}_2 + \tilde{\boldsymbol{Q}}\tilde{\boldsymbol{Z}}_1. \quad (17)$$

To achieve the degrees of freedom $(M_1N_1, M_2(N_1-M_1))$ for both users, it is sufficient to design our $\tilde{\boldsymbol{P}}$ and $\tilde{\boldsymbol{Q}}$ to satisfy the following conditions simultaneously

1) rank$(\tilde{\boldsymbol{U}}) = M_1N_1$.
2) rank$(\tilde{\boldsymbol{P}}) = M_2(N_1-M_1)$.
3) $\tilde{\boldsymbol{V}} = \boldsymbol{0}$.

The second condition can be easily satisfied. Because rank$(\tilde{\boldsymbol{P}})$ = rank$(\boldsymbol{P})$rank$(\boldsymbol{I}_{M_2})$, we only need to design $\boldsymbol{P}$ such that rank$(\boldsymbol{P}) = N_1 - M_1$. As to the third condition, notice that

$$\begin{aligned}\tilde{\boldsymbol{V}} &= (\boldsymbol{Q}\otimes\boldsymbol{I}_{N_1})(\boldsymbol{I}_{N_1}\otimes\boldsymbol{H}_{12}(1))(\boldsymbol{P}\otimes\boldsymbol{I}_{M_2}) && (18)\\ &= (\boldsymbol{Q}\boldsymbol{I}_{N_1}\boldsymbol{P})\otimes(\boldsymbol{I}_{N_1}\boldsymbol{H}_{12}(1)\boldsymbol{I}_{M_2}) && (19)\\ &= \boldsymbol{Q}\boldsymbol{P}\otimes\boldsymbol{H}_{12}(1). && (20)\end{aligned}$$

It is therefore sufficient (and also necessary) to have $\boldsymbol{Q}\boldsymbol{P} = 0$. Then the key problem is to find a $\boldsymbol{Q}$ such that

$$\tilde{\boldsymbol{U}} = (\boldsymbol{Q}\otimes\boldsymbol{I}_{N_1})\tilde{\boldsymbol{H}}_{11} \quad (21)$$

has full rank M_1N_1. The matrix $\tilde{\boldsymbol{U}}$ is of size $M_1N_1 \times M_1N_1$ and has the following structure

$$\tilde{\boldsymbol{U}} = \begin{bmatrix} q_{11}\boldsymbol{H}_{11}(1) & q_{12}\boldsymbol{H}_{11}(2) & \cdots & q_{1N_1}\boldsymbol{H}_{11}(N_1)\\ q_{21}\boldsymbol{H}_{11}(1) & q_{12}\boldsymbol{H}_{11}(2) & \cdots & q_{2N_1}\boldsymbol{H}_{11}(N_1)\\ \vdots & \vdots & \ddots & \vdots\\ q_{M_11}\boldsymbol{H}_{11}(1) & q_{M_12}\boldsymbol{H}_{11}(2) & \cdots & q_{M_1N_1}\boldsymbol{H}_{11}(N_1)\end{bmatrix}.$$

To show that $\tilde{\boldsymbol{U}}$ has full rank, we use an idea of [24] and need the following lemma.

Lemma 4: [24, Lemma 1] Consider an analytic function $f(\boldsymbol{x})$ of several variables $\boldsymbol{x} = [x_1,\ldots,x_n]^T$. If f is nontrivial in the sense that there exists $\boldsymbol{x}_0 \in \mathbb{C}^n$ such that $f(\boldsymbol{x}_0) \neq 0$, then the zero set of $f(\boldsymbol{x})$ Z $:= \{\boldsymbol{x} \in \mathbb{C}^n | f(\boldsymbol{x}) = 0\}$ is of measure (Lebesgue measure in $\mathbb{C}^n$) zero. ■

Because the determinant of $\tilde{\boldsymbol{U}}$ is an analytic polynomial function of elements of $\boldsymbol{H}_{11}(t), t = 1,\ldots,N_1$, we only need to find a specific pair of $\boldsymbol{Q}$ and $\boldsymbol{H}_{11}(t), t = 1,\ldots,N_1$ such that $\tilde{\boldsymbol{U}}$ is full rank. We propose the following:

$$q_{mn} = \exp\left(-j\frac{2\pi(m-1)(n-1)}{N_1}\right)$$
$$m = 1,\cdots,M_1 \quad n = 1,\ldots,N_1. \quad (22)$$

Let $W := \exp(-j2\pi/N_1^2)$. Take the realizations of $\tilde{\boldsymbol{H}}_{11}(t)$, $t = 1,\ldots N_1$ as shown in (23).

It can be verified that for such choices of $\boldsymbol{Q}$ and $\boldsymbol{H}_{11}(t)$, $\tilde{\boldsymbol{U}}$ is a Vandermonde matrix of different columns, and hence of full rank. We also notice that $\tilde{\boldsymbol{U}}$ is a leading principal minor of a permuted fast Fourier transform (FFT) matrix with size $N_1^2 \times N_1^2$. The permutation is as follows: Index the columns of an FFT matrix $0,1,\ldots,N_1^2-1$, and then permute them in an order shown below:

$$(0, N_1, 2N_1, \ldots, (M_1-1)N_1),$$
$$(1, N_1+1, 2N_1+1, \ldots, (M_1-1)N_1+1), \ldots$$

Based on Lemma 4, if we choose the nulling matrix using $\boldsymbol{Q}$ as specified in (22), $\tilde{\boldsymbol{U}}$ has full rank almost surely. One choice of the corresponding $\boldsymbol{P}$ matrix with respect to (22) is the following

$$p_{nm} = \exp\left(j\frac{2\pi(m-1)(n-1)}{N_1}\right),$$
$$n = 1,\ldots,N_1, \quad m = M_1+1,\cdots,N_1. \quad (24)$$

We note that the matrix $[\boldsymbol{Q}^H \ \ \boldsymbol{P}]$ is an FFT matrix. Our choices of $\boldsymbol{Q}$, $\boldsymbol{P}$ in (22) and (24) have a frequency domain interpretation. The signal of user 2 is transmitted over frequencies corresponding to the last $N_1 - M_1$ columns of an FFT matrix, whereas the first user's signal is transmitted on all frequencies. We also point out that when $N_1/M_1 = L \in \mathbb{Z}$, it is possible to achieve the same DoF using only L fold time expansion. Due to space limit, we will report that elsewhere. The previous results can be summarized as:

Lemma 5: For the two-user MIMO Z interference channel without CSIT, with antenna numbers $M_1 < N_1 < \min(M_2, N_2)$. User 1 and user 2 can achieve DoF pair $(M_1, \min(M_2,N_2)(N_1-M_1)/M_2)$. ■

The achievability of Lemma 5 is based on time expansion, antenna mode switching at transmitter 1, with jointly designed beamforming matrix at transmitter 2 and nulling matrix at receiver 1. We end the discussion of MIMO ZIC with the following theorem:

Theorem 2: The DoF region of two-user MIMO Z interference channel without CSIT is described by the inequalities (10) and (11) if transmitter 1 has the antenna mode switching ability.

Proof: The achievability when $N_1 \geq N_2$ can be established by Lemma 2. When $M_1 < N_1 < \min(M_2, N_2)$, the achievability follows from Lemma 5. The achievability of the remaining cases is based on the achievability of FIC given in [17]. ■

C. Two-User MIMO FIC without CSIT

Finally, we have the following result.

Theorem 3: The DoF region of two-user MIMO full interference channel without CSIT is described by the

$$\boldsymbol{H}_{11}(t)=\begin{bmatrix} W^0 & W^0 & \cdots & W^0 \\ W^{t-1} & W^{N_1+t-1} & \cdots & W^{(M_1-1)N_1+t-1} \\ \vdots & \vdots & \ddots & \vdots \\ W^{(N_1-1)(t-1)} & W^{(N_1-1)(N_1+t-1)} & \cdots & W^{(N_1-1)[(M_1-1)N_1+t-1]} \end{bmatrix}_{N_1\times M_1} \tag{23}$$

inequalities (6), (7) and (8) if transmitter 1 has the antenna mode switching ability.

Proof: When $M_1 < N_1 < \min(M_2, N_2)$, we can achieve $(M_1, \min(M_2, N_2)(N_1 - M_1)/M_2)$ in the ZIC with same number of antennas. Hence, based on Lemma 3, it is also achievable in FIC. The achievability of the remaining cases is given in [17]. ■

V. Conclusions

We derived a few results on the exact DoF region for the MIMO Z and full interference channels with perfect channel state information at receiver, including results for i) the Z interference channel with and without channel state information at the transmitter, and ii) the full interference channel without channel state information at the transmitter. The achievability scheme we designed for the case of $M_1 < N_1 < \min(M_2, N_2)$ relies on time expansion, antenna mode switching at transmitter one, and has a frequency domain interpretation. By combining our results with previously known results, we completely characterized the DoF regions for both Z and full interference channels when transmitter antenna mode switching is allowed. We comment that when antenna mode switching is not allowed, the problem of DoF regions for these channels are still not completely resolved.

References

[1] A. Carleial, “A case where interference does not reduce capacity (Corresp.),” *IEEE Trans. Inform. Theory*, vol. 21, no. 5, pp. 569–570, May 1975.

[2] H. Sato, “The capacity of the Gaussian interference channel under strong interference (Corresp.),” *IEEE Trans. Inform. Theory*, vol. 27, no. 6, pp. 786–788, June 1981.

[3] A. Gamal and M. Costa, “The capacity region of a class of deterministic interference channels (Corresp.),” *IEEE Trans. Inform. Theory*, vol. 28, no. 2, pp. 343–346, Feb. 1982.

[4] R. H. Etkin, D. N. C. Tse, and H. Wang, “Gaussian interference channel capacity to within one bit,” *IEEE Trans. Inform. Theory*, vol. 54, no. 12, pp. 5534–5562, Dec. 2008.

[5] X. Shang, G. Kramer, and B. Chen, “Outer bound and noisy-interference sum-rate capacity for symmetric Gaussian interference channels,” in *Information Sciences and Systems, 2008. CISS 2008. 42nd Annual Conference on*, 2008, pp. 385–389.

[6] V. Sreekanth Annapureddy and V. Veeravalli, “Sum capacity of the Gaussian interference channel in the low interference regime,” in *Information Theory and Applications Workshop, 2008*, 2008, pp. 422–427.

[7] A. Motahari and A. Khandani, “Capacity bounds for the Gaussian interference channel,” in *Proc. IEEE Intl. Symp. on Info. Theory*, 2008, pp. 250–254.

[8] G. Bresler and D. Tse, “The two-user Gaussian interference channel: a deterministic view,” *European Trans. on Telecomm.*, vol. 19, no. 4, pp. 333–354, Apr. 2008.

[9] G. Bresler, A. Parekh, and D. Tse, “The approximate capacity of the many-to-one and one-to-many gaussian interference channels,” 2008. [Online]. Available: http://arxiv.org/abs/0809.3554v1

[10] S. Avestimehr, S. Diggavi, and D. Tse, “Wireless network information flow: a deterministic approach,” 2009. [Online]. Available: http://arxiv.org/abs/0906.5394

[11] V. S. Annapureddy and V. V. Veeravalli, “Sum capacity of MIMO interference channels in the low interference regime,” 2009. [Online]. Available: http://arxiv.org/abs/0909.2074

[12] X. Shang, B. Chen, G. Kramer, and H. V. Poor, “Capacity regions and sum-rate capacities of vector Gaussian interference channels,” 2009. [Online]. Available: http://arxiv.org/abs/0907.0472

[13] A. Goldsmith, S. Jafar, N. Jindal, and S. Vishwanath, “Capacity limits of MIMO channels,” *IEEE Journal on Selected Areas in Communications*, vol. 21, no. 5, pp. 684–702, May 2003.

[14] V. Cadambe and S. Jafar, “Interference alignment and degrees of freedom of the K user interference channel,” *IEEE Trans. Inform. Theory*, vol. 54, no. 8, pp. 3425–3441, Aug. 2008.

[15] S. Jafar and M. Fakhereddin, “Degrees of freedom for the MIMO interference channel,” *IEEE Trans. Inform. Theory*, vol. 53, no. 7, pp. 2637–2642, July 2007.

[16] S. Jafar and S. Shamai, “Degrees of freedom region of the MIMO X channel,” *IEEE Trans. Inform. Theory*, vol. 54, no. 1, pp. 151–170, Jan. 2008.

[17] C. Huang, S. Jafar, S. Shamai, and S. Vishwanath, “On degrees of freedom region of MIMO networks without CSIT,” 2009. [Online]. Available: http://arxiv.org/abs/0909.4017

[18] Y. Zhu and D. Guo, “Isotropic MIMO interference channels without CSIT: the loss of degrees of freedom,” 2009. [Online]. Available: http://arxiv.org/abs/0910.2961

[19] C. S. Vaze and M. K. Varanasi, “The degrees of freedom regions of MIMO broadcast, interference, and cognitive radio channels with no CSIT ,” 2009. [Online]. Available: http://arxiv.org/abs/0909.5424

[20] X. Shang, B. Chen, G. Kramer, and H. V. Poor, “MIMO Z-interference channels: capacity under strong and noisy interference,” 2009. [Online]. Available: http://arxiv.org/abs/0911.4530

[21] Y. Zhu and D. Guo, “Ergodic fading one-sided interference channels without state information at transmitters,” 2009. [Online]. Available: http://arxiv.org/abs/0911.1082

[22] S. A. Jafar, “Exploiting channel correlations - simple interference alignment schemes with no CSIT,” 2009. [Online]. Available: http://arxiv.org/abs/0910.0555

[23] C. Wang, T. Gou, and S. A. Jafar, “Aiming perfectly in the dark — blind interference alignment through staggered antenna switching,” 2010. [Online]. Available: http://arxiv.org/abs/1002.2720

[24] N. Sidiropoulos and R. Budampati, “Khatri-Rao space-time codes,” *Signal Processing, IEEE Transactions on*, vol. 50, no. 10, pp. 2396–2407, Oct. 2002.